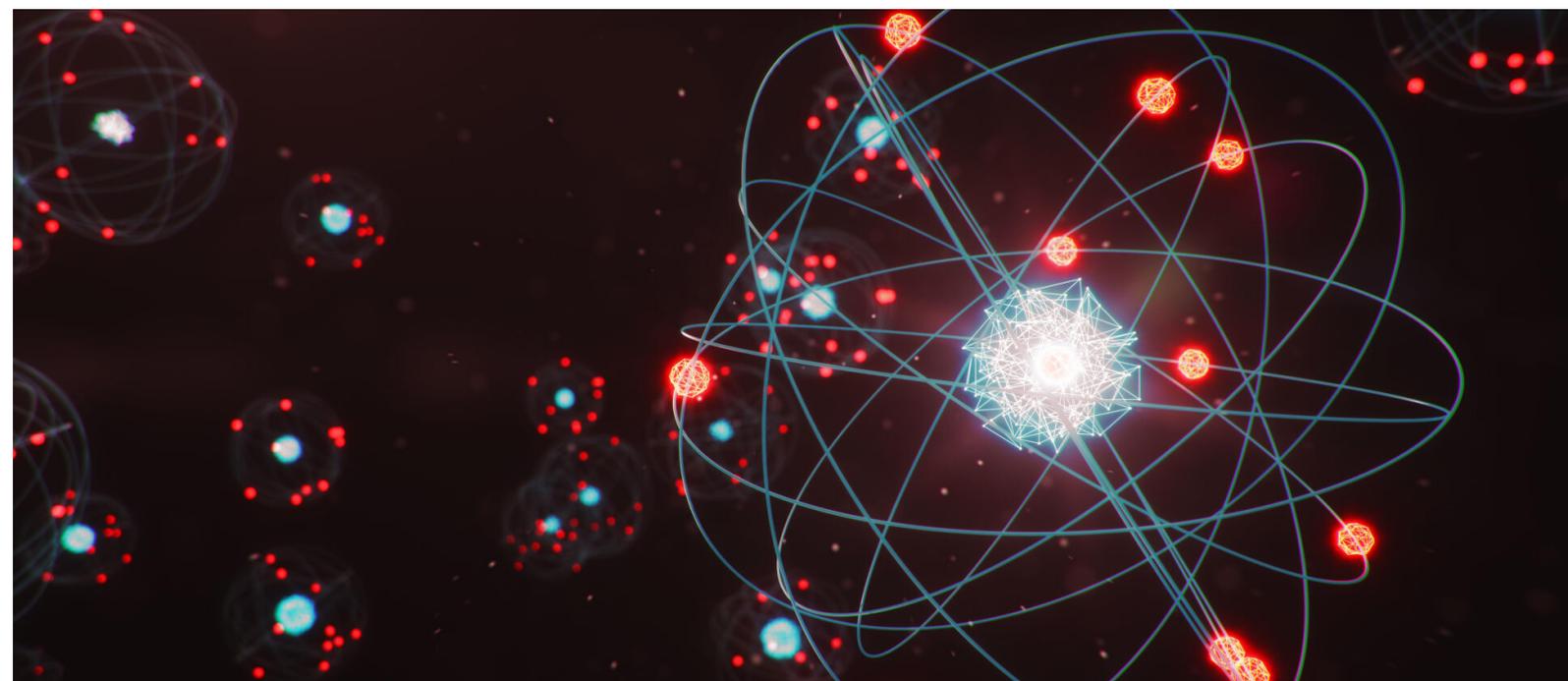


# NUCLEAR PHYSICS

## UNIT- I



**BY: DR. MANOJ SINGH  
DEVA NAGRI COLLEGE, MEERUT.**

## Unit-1 Introductory concept of nuclei

### Nuclear sizes

The Rutherford  $\alpha$ -scattering experiment established that the mass of an atom is concentrated within a small, positively charged region at the centre which is called the nucleus of an atom.

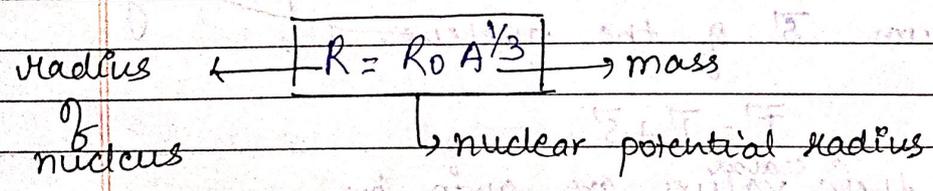
An electron interacts with a nucleus only through electric forces while a neutron interacts only through nuclear forces. Thus, electron scattering tells the distribution of charge in a nucleus and neutron scattering tells the distribution of nuclear mass.

The volume of a nucleus is directly proportional to the no. of nucleons in it, which is its mass no.  $A$ .

Let  $R$  be the radius of nucleus  
then, volume of nucleus =  $\frac{4}{3} \pi R^3$

So,  $R^3 \propto A$   
 $R \propto A^{1/3}$

OR



where  $R_0$  is known as nuclear potential radius  
 $R_0 = 1.5 \times 10^{-15} \text{ m}$  or  $1.5 A^{1/3} \text{ fm}$

$$1 \text{ f} = 10^{-15} \text{ m} = 10^{-5} \text{ \AA} = 10^{-18} \text{ cm}$$

density  $\rho = 1.07 \text{ A}^3$

### Nuclear Angular momentum

An atomic nucleus is made up of protons and neutrons, collectively called the nucleons. In 1924, W. Pauli while explaining the hyperfine structure of spectral lines, suggested that certain atomic nuclei may possess an intrinsic angular momentum as well as magnetic moment. The nuclear angular momentum can be deduced from measurement of the multiplicity and relative spacing of the spectral lines.

It has been found that each of the protons and neutrons has an intrinsic spin angular momentum ( $= \frac{1}{2} \hbar$ ), and an orbital angular momentum about the centre of mass of the nucleus. The total angular momentum of the nucleus  $\vec{I}$  is the vector sum of the orbital angular momentum  $\vec{L}$  and the spin angular momentum  $\vec{S}$  of the nucleus.

i.e. 
$$\vec{I} = \vec{L} + \vec{S}$$

where their scalar values are given by

$$I = \sqrt{I(I+1)} \hbar$$

and 
$$L = \sqrt{L(L+1)} \hbar$$

$$S = \sqrt{S(S+1)} \hbar$$

further 
$$\vec{S} = \sum \vec{s}_i$$
  

$$I = \sum \vec{I}_i$$

where  $\vec{s}_i$  and  $\vec{I}_i$  are the spin angular momentum and orbital angular momentum of the individual nucleons. Since,  $\vec{s}_i = \frac{1}{2} \hbar$ ,  $s$  can be either integer or half integer depending on whether the no. of nucleons  $N$  in the nucleus is even or odd.

The magnitude 'I' of the total nuclear angular momentum is called nuclear spin. Measurements of the ground state spin of the nuclei give following information regarding nuclear spin.

- For even Z of even A nuclei, the value of I in the ground state is always zero.
- For odd Z, <sup>odd</sup> even A nuclei, then I is in integral
- For even Z & odd A nuclei or odd Z & even A, value of I is in half integral lying b/w  $\frac{1}{2} \hbar$  to  $\frac{5}{2} \hbar$ .

## Nuclear magnetic dipole moment

Any charged particle moving in a closed path produces a magnetic field, which at larger distances act as a magnetic dipole. Hence, the magnetic dipole moment is defined by the strength of poles (north and south) as taken with the consideration, then the product of intermolecular separation by the two poles and the magnitude of strength of magnetic field define its magnetic dipole. Each nucleus contain nuclear dipole moment when the motion of nucleus is considered.

Let us consider a particle having charge 'q', and mass 'm' which rotates about a central axis with frequency 'v' then the current given by the charged particle is  $i = qv$   $\rightarrow (1)$

We know that, the central force of an atom is define by 'Kepler's law' according to which "any classical particle sweep out equal area in equal time interval". In other words, the areal velocity of the particle remain constant i.e.

$$\frac{dA}{dt} = \text{constant}$$

In the form of angular momentum, above eq<sup>n</sup> can be written as

$$\frac{dA}{dt} = \frac{1}{2m} \rightarrow \text{total angular momentum}$$

On integrating, we get total area over one period

$$A = \frac{1}{2} \pi r^2 \rightarrow (2)$$

The magnetic momenta of a ring containing area A can be expressed as

$$M_e = iA$$

$$M_e = (qv) \left( \frac{1}{2} \pi r^2 \right)$$

$$M_e = \frac{q}{2m} \pi r^2 \quad (v = \frac{r}{T})$$

This eq<sup>n</sup> is described in the form of orbital angular momenta 'l' total angular momenta. Now a correction factor 'g' is introduced in above eq<sup>n</sup> which explain the motion of the particle if replace  $q = e$

$$M_e = g_e \left( \frac{e}{2m} \right) \pi r^2 \rightarrow (3)$$

Similarly, for spin component the above eq<sup>n</sup> can be written as

$$M_s = g_s \left( \frac{e}{2m} \right) \pi r_s^2 \rightarrow (4)$$

eg (13) & (14) represent the magnetic moment  
 will form a total magnetic moment can  
 Now, the total magnetic moment can  
 be expressed as

$$\mu = \mu_L + \mu_S$$

$$\mu = \sum_{i=1}^A \left[ \frac{q_i e}{2m_i} L_i + g_i \frac{e}{2m_i} S_i \right]$$

$$\mu = \frac{q e}{2m} [L + g S]$$

where  $L + g S \rightarrow$  total angular momentum  
 and  $\frac{e}{2m}$  gives the dimensionless ratio  
 and  $g =$  gyromagnetic ratio

Now, total angular momentum of the  
 nucleus

$$I = \sum_{k=1}^A L_k + \sum_{k=1}^A S_k$$

where

$$\mu = \frac{q e}{2m} I$$

$$\text{Or } \mu = g \frac{e \hbar}{2m} \frac{I}{\hbar}$$

When  $I$  is measured in terms of  $\hbar$   
 then it is called angular momentum  
 quantum number.

Using  $[1(1+1)]^{1/2}$  or  $[5(5+1)]^{1/2}$  instead of  $L$ 's,

We get

$$g = \frac{1}{2} (g_L + g_S) + \frac{1}{2} (g_L - g_S) \frac{L(L+1) - S(S+1)}{I(I+1)}$$

Magnetic dipole moment in terms of magneton

$$\mu_N = \frac{e \hbar}{2mp} = 5.05 \times 10^{-27} \text{ Am}^2$$

$$= 3.15 \times 10^{-8} \text{ eV} \cdot \text{meter}^{-1}$$

Magnetic dipole moment in terms of Bohr  
 magneton

$$\mu_B = \frac{e \hbar}{2me}$$

then

$$\frac{\mu_N}{\mu_B} = \frac{me}{mp} = \frac{1}{1836}$$

Therefore, nuclear magneton is 1836 times  
 smaller than Bohr magneton.

### Electric quadrupole moment

The electric quadrupole moment measures the  
 separation of nucleons from the spherical symmetrical  
 shape. It means that when an atom is placed  
 in electric field then, due to the electric force  
 nucleus begin to vibrate about their mean position.  
 Hence, the path of the particle should be harmonic  
 in nature. It means that the nucleus inside  
 the nucleus behave as harmonic oscillator.

and displacement of charged particle is obtained and displacement vector at momenta known as this displacement vector inside a nucleus. Electrical moments outside a nucleus. nucleus of charge particle does not represent in stationary position nucleus does not represent any dipole moment. But, when the nucleus starts to rotate in field, when the nucleus starts to rotate in stationary position, doesn't produce any moment because of vector of mass or vector of charge but when it is placed in electric field it produces a momentum known as electric quadrupole moment.

The electric quadrupole moment of a nucleus is calculated as follows from the classical considerations. Let us consider that the charge is not distributed at the center of the nucleus (departure from spherical symmetry) but is located at  $P'$  having rectangular coordinates  $(x, y, z)$ . The potential at a point  $P$  on the  $z$ -axis due to this charge is

$$\phi_P = \frac{1}{4\pi\epsilon_0} \frac{e}{a_1} \quad \text{--- (1)}$$

but  $a_1 = (a^2 + z^2 - 2az \cos\alpha)^{1/2}$  --- (2)

$a$  = distance of the charge from the origin and is given by

$$a^2 = (x^2 + y^2 + z^2)^{1/2} \quad \text{and } \cos\alpha = \frac{z}{a}$$

Eq (1) becomes

$$\phi_P = \frac{1}{4\pi\epsilon_0} \frac{e}{(a^2 + z^2 - 2az \cos\alpha)^{1/2}}$$

$$\phi_P = \frac{e}{4\pi\epsilon_0 a} \left( 1 - 2 \frac{z \cos\alpha}{a} + \frac{z^2}{a^2} \right)^{-1/2}$$

--- (3)

$$\phi_P = \frac{1}{4\pi\epsilon_0} \left[ \frac{e}{a} + \frac{e}{a^2} z \cos\alpha + \frac{e}{a^3} z^2 \left( \frac{3 \cos^2\alpha - 1}{2} \right) + \dots \right]$$

$$\text{Or } \phi_P = \frac{1}{4\pi\epsilon_0} \sum_{n=0}^{\infty} \frac{e z^n}{a^{n+1}} P_n(\cos\alpha) \quad \text{--- (4)}$$

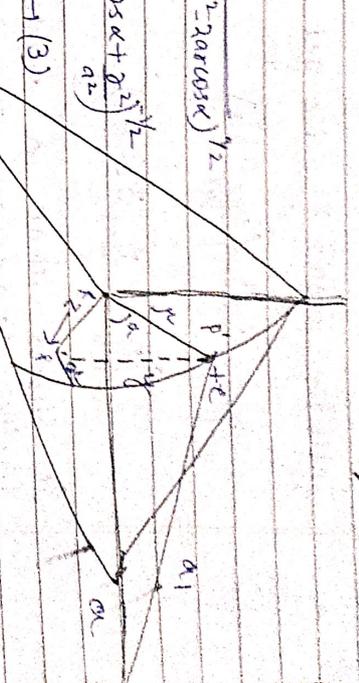
where  $P_n(\cos\alpha)$  are Legendre Polynomials &  $n$  is the multipole order.

In eq (4) the coeff. of  $\frac{1}{a}$  is known as monopole strength, the coeff. of  $\frac{1}{a^2}$  is the  $z$ -component of

quadrupole moment, the coeff. of  $\frac{1}{a^4}$  is the  $z$ -component of octupole moment etc.

First term is Coulomb potential. Thus, the nucleus possess a net electric quadrupole moment given by

$$Q = e z^2 \left( \frac{3 \cos^2\alpha - 1}{2} \right) = \frac{e z^2}{2} (3 \cos^2\alpha - 1)$$



Putting  $\rho(r) = \frac{Z}{4\pi R^3}$ , we get

$$Q = \frac{e}{4\pi} \int \left( \frac{3z^2}{R^2} - 1 \right) d\tau$$

$$Q = \frac{e}{4\pi} \int \left( \frac{3z^2}{R^2} - 1 \right) d\tau \rightarrow (6)$$

It shows that a nucleus will possess a finite quadrupole moment only when the protons are placed, asymmetrically. If the protons are symmetrically arranged to three axes, the net quadrupole moment is zero.

Let  $x = y = z = 0$  then,

$$Q = \frac{e}{4\pi} \int (3R^2 - R^2) d\tau$$

$$Q = eR^2$$

For uniformly charged ellipsoid of revolution defined by the equations

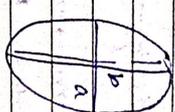
$$\frac{x^2 + y^2}{a^2} + \frac{z^2}{b^2} = 1$$

$$Q = \frac{Ze}{5} (b^2 - a^2)$$

$a > b$  = oblate  
 $a < b$  = prolate  
 $a = b$  = spherical

For a charge distribution (prolate spheroid) stretched in a z direction (prolate spheroid)  $Q$  is positive while for a charge distribution flattened in the z direction (oblate spheroid)  $Q$  is negative.

$Q$  is negative.



$Q > 0$   
prolate



$Q < 0$   
oblate

The dimensions of the quadrupole moment  $Q$  is measured in  $10^{-28} \text{ m}^2$

### Parity Quantum number

The parity of a system refers to the behaviour of a wavefunction w.r.t. under inversion of co-ordinates to the origin.

When  $x$  is replaced by  $-x$ ,  $y$  by  $-y$  and  $z$  by  $-z$ , if the change of sign of  $\psi(x, y, z)$  does not change the sign of wavefunction i.e.

$$\psi(x, y, z) = \psi(-x, -y, -z)$$

then, the wavefunction is said to have positive or even parity. And if the change of sign of  $\psi(x, y, z)$ , changes the wavefunction i.e.

$$\psi(x, y, z) = -\psi(-x, -y, -z)$$

then, the wavefunction is said to have negative parity.

or odd parity

$$\psi(x, y, z) = P(-x, -y, -z)$$

In general,  $\psi(x, y, z) = P(-x, -y, -z)$   
where  $P = \pm 1$   
 $P$  can be taken as a quantum no. of the property defined by it is called the parity of the system.

Parity is related to the orbital quantum no.  $l$  by the relation

$$P = (-1)^l$$

It means that the orbital angular momentum

define the parity i.e. even or odd.

Nuclei in the form of nucleons the parity is

define as  $P = (-1)^{n_l} + p_e$

where  $n$  &  $l$  represent no. of neutron & proton respectively.

### Statistics of nuclear particles

According to the classification of nuclear particles

we know that the nuclear particles are divided into two parts known as fermions or antisymmetric particles and bosons or symmetric particles. These particles are classified on the basis of their spin and motion.

of the particle is half integral spin then the spin is integral then it is boson. Again on the basis of the motion it can be explained as on changing the state of the particle, if

the wave function remains same then particle is symmetric particle or bosons and on changing the state of the particle if wave function changes then particle is antisymmetric or fermions.

In mathematical form these particles are defined by considering the state (or position) of the particle

if  $\psi(x, y)$  is symmetric then,  
 $\psi(x, y) = \psi(y, x)$  (particle is symmetric)

and  
 $\psi(x, y) = -\psi(y, x)$  (particle is antisymmetric)

According to these classifications the two nuclear statistics obtained known as

- Bose Einstein statistics or symmetrical statistics.
- Fermi Dirac statistics or antisymmetrical statistics.

### Bose Einstein statistics or B.E statistics

The distribution of energy for bosons particles is explained by  $\psi(x, y)$  and B.E statistics. According to which equal amount of energy is distributed among the various energy state of Bose particle.

This statistics is only applicable for those particles which consist integral spin and does not follow Pauli's exclusion principle i.e. this statistics is applicable for those particles which define the relative probability for the same state.

In other words, we can say that B.E statistics is applicable for those particles which have periodic motion. It means that the periodicity of particle is also described by B.E statistics

In mathematical form, it can be expressed as -

$$n_i = \frac{g_i}{e^{kT(E_i - \mu)} + 1}$$

where

$n_i$  = no. of particle

$g_i$  = no. of state

$k$  &  $\beta$  = two parameters

$E_i$  = energy of the different particles.

In the case of B.E statistics, the particles remain in its original position whose wavefunction does not overlap to each other i.e. the condition of indistinguishability is obtained in such statistics.

### Fermi Dirac Statistics or F.D statistics

This statistics is used to explain the distribution of energy for fermi particles.

According to this statistics, equal amount of energy is distributed among the various energy state of fermi particles.

This statistics is applicable for those particles which consist half integral spin and follow Pauli's exclusion principle. i.e. this statistics is applicable for those whose motion changes during excitation. Also for non-periodic motion.

In mathematical form it can be expressed as.

$$n_i = \frac{g_i}{e^{kT(E_i - \mu)} - 1}$$

From above we can say that the fermi dirac statistics consist zero probability in different quantum state at the same time.

### Isospin (Isotopic spin, Isobari spin) quantum number

Isospin is a quantum number related to the strong interactions. Later on they are referred as Isobari because neutron and proton have same masses. Isospin is a quantum number which define the state of the particle. It has mathematical properties similar to those of spin, but it has no direct physical relationship to spin.

A set of Isospin has  $(2I+1)$  states. The quantum number  $I_3$  or  $I_z$  called the third component of isospin  $I$ , has varying values from  $-I$  to  $I$ . The component  $I_3$  is related with the no. of neutrons  $N$  and no. of proton  $Z$  for the particular isobars as

$$I_3 = \frac{1}{2}(N - Z)$$

Thus the possible values of  $I_3$  are  $\frac{1}{2}$  for  $N$  and  $-\frac{1}{2}$  for  $Z$ .

Isobari spin is conserved in nuclear interactions in the same manner as the conservation of total nuclear angular momentum.